Model 'Pencil' Rocket – 100 ft. Range

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This primary goal of this project is to use a given model rocket and insert the necessary payload to ensure that the rocket achieves a height of 100 ft. In order to do this the given rocket is analyzed and its dimensions are measured. Relevant drag calculations are conducted and the B6-4 engine of the rocket is analyzed to determine specific thrust. The relevant data is compiled and processed for the rocket flight duration and a payload estimate is made in order to achieve the given height. The experimental launch was successful and the experimental height was close to the required mission parameters. The rocket's moment of inertia about the central axis was calculated as well as its center of gravity. A scaled drawing of the rocket was also designed.

I) NOMENCLATURE

C_{D_O} = Profile Drag Coefficient
$C_{D_{BT}}$ = Body Tube Drag Coefficient
C_{D_B} = Base Drag Coefficient
C_{D_F} = Fin Unit Drag Coefficient
C_f = Turbulent Flat Plate Skin Friction Coefficient
λ_B = Length of the Rocket Body
d = Maximum Diameter of Rocket Body
d_B = Base Diameter
I_{sp} = Specific Impulse
S_B = Maximum Body Frontal Area
S_S = Wetted Area of Rocket Body
T = Thrust
v_b = Burnout Velocity
m = Mass (initial *i*, final *f*)
z = Center of gravity height from base

II) INTRODUCTION

The given rocket was the Estes® 'Sky Writer™' – No. 2 and the engine used was the B6-4. The mission parameters required inserting a precalculated payload in the rocket in order to ensure that the rocket reaches a height of 100 ft. This height was measured experimentally using angle measurements from fixed distances from the launch point. In order to determine the required payload, a number of calculations were performed. The details of these calculations are included in the Method section. Firstly, the drag acting on the rocket during flight was calculated. This was done by summing various drag contributions and calculating the profile drag coefficient C_{Do}. This was used to calculate the drag contribution for the duration of the flight. A separate experiment was conducted in order to determine the specific thrust of the engines used I_{sp}. This value was used in order to determine the rocket burnout velocity, i.e., the velocity of the rocket at the time the engine burns out. The burnout velocity was further utilized with equations of motion for the rocket and energy equations in order to determine an appropriate payload that must be inserted in the rocket. Calculations were also done to determine the center of mass of the rocket and the mass moment of inertia. ProEngineeerWildfire® was used to create a scaled drawing.

III) METHOD

The drag coefficient for the body tube $C_{D_{BT}}$ can be found using the equation:

$$C_{D_{BT}} = C_f \left[1 + \frac{60}{(\lambda_B / d)^3} + 0.0025(\lambda_B / d) \right] \frac{S_S}{S_B} \qquad (1)$$

This value can also be found using Shevell's method, which is known to be:

$$C_{D_{BT}} = \frac{K_i C_f S_{wet}}{S_{ref}} \qquad (2)$$

The base drag coefficient C_{D_B} can be found using the equation:

$$C_{D_B} = 0.029 \left(\frac{d_B}{d} \right)^3 / \sqrt{C_{D_{BT}}} \qquad (3)$$

The fin unit drag coefficient C_{D_F} can be found using Shevell's methods for aircraft wings, where the reference area is equal to the maximum body frontal area.

The total drag coefficient for the rocket can be found by adding the coefficients of drag above and is given by,

$$C_D = 1.05\ (C_{D\,bt} + C_{Db} + C_{Df}) \qquad (4)$$

The moment of inertia can be calculated using the known equation:

$$I = \sum_{i=1,N} m_i \left| k \times r_i \right|^2 \qquad (5)$$

The letter 'k' represents the kinetic energy (½mv²).

The center of gravity for the rocket can be calculated using a static equation:

$$\bar{z} = \sum_{i}^{n} \frac{m_i z_i}{m_i} \tag{6}$$

The mass of each rocket component was recorded and the center of gravity for the whole assembly was found.

The velocity of the rocket when the engine burns v_b out must be found in order to calculate important data while the rocket is in flight. First, the thrust must be found.
Thrust is a force, therefore, according to Newton's second law of motion:

$$T = m\frac{dv}{dt} \tag{7}$$

The specific impulse can be multiplied

$$\dot{W} I_{sp} = \dot{m} g I_{sp} \tag{8}$$

Mass flow is found to be:

$$\dot{m} = \frac{-dm}{dt} \tag{9}$$

Thrust can now be expressed as:

$$T = -g_o I_{sp} \frac{dm}{dt} \tag{10}$$

The burnout velocity can then be found setting thrust equal to a force, and integrating.

$$\int_{m}^{m_f} \frac{-dm}{m} = \int_{0}^{v_b} \frac{dv}{g_o I_{sp}} \tag{11}$$

$$v_b = g_o I_{sp} \ln\left(\frac{m}{m_f}\right) \tag{12}$$

IV) THRUST DATA

The thrust generated by the B 6-4 engine is characterized by the specific thrust impulse data which is essentially the Impulse delivered by the engine over its operational time per unit propellant mass. In order to determine this cantilever beam experiment was performed in which the rocket exerted thrust generated a voltage due to beam deflection and this was measured over time.

Figure 1(Engine Static Test)

This voltage was then converted into a thrust factor by suitable scaling using the slope of the graph to follow.

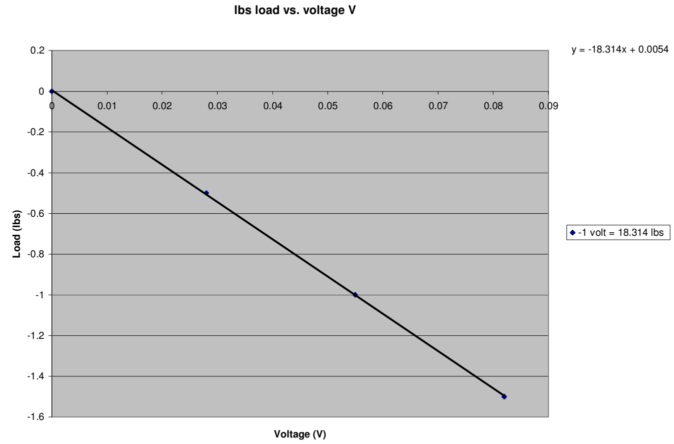

Figure 2 (Scaling Graph for cantilever beam)

The area under the graph was calculated in order to obtain the total thrust and this was divided by the propellant mass of 6.4 grams in order to determine the value of I_{SP}.

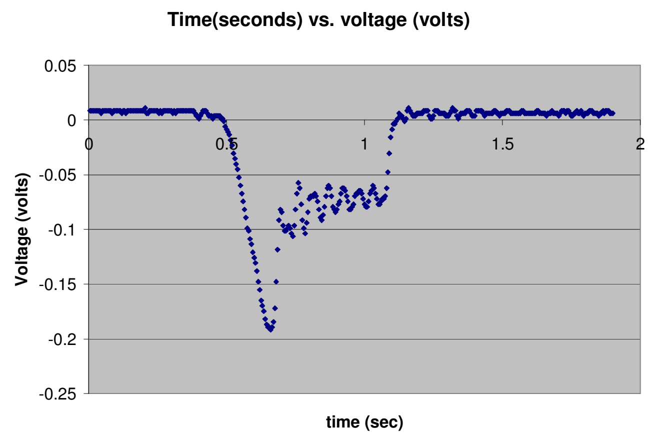

Figure 3 (Isp determination from the graph)

Thus, the area under the graph was found to be 0.054 Volts.sec that corresponds to 0.92 lbs/sec. Dividing this by the payload we have a specific impulse of 73.71 lb.s/lbm.

V) CALCULATIONS

In this section, the major calculations of the experiment are outlined and the main rocket characteristic, i.e. the payload, is determined.

1. **Drag Calculation:**

   Using equation 1 from the method, $C_{D_{BT}}$ can be obtained:

$$C_{D_{BT}} = .002\left[1+\frac{60}{18674}+0.0025(26.53)\right]\frac{0.55}{.0052}$$

$$= .226$$

Using Shevell's method (equation 2), the body tube drag coefficient is found to be:

$$C_{D_{BT}} = .158 \quad (13)$$

This shows a 30% difference in values.

Equation 3, is used to find that,

$$C_{D_B} = .0305 \quad (14)$$

C_{D_F} can be found by taking the wetted area of the fins and accounting for the curvature.

$$C_{D_F} = 0.0056\times 4\times 1.02 = 0.0206 \quad (15)$$

Using the coefficients found above, the total drag coefficient can be found using equation 4 to be ***0.2771***.

2. Center of Mass Calculation:

Figure 4 (Assembled Rocket)

The center of mass is calculated using equation 5. The mass of various components and the locations of their individual centers of gravity are summarized in the table below,

Table 1(CG calculation)

Part	Mass (g)	CG location (in)	m x CG
Base	12.7	1.3	16.51
Lower body tube	6.9	5.8	40.02
Upper body tube	6	15.55	93.3
Engine mount tube	1.2	1.374	1.6488
Engine	18	1.374	24.732
Nose cone	7.5	16.5	123.75
Parachute	3.2	16.5	52.8
Adaptor ring	3.8	2.2	8.36
Red coupler	1.2	11.7	14.04
Payload	100	4.2	420

The heights shown in this table are shown as measurements taken from the base of the rocket. The CG for the rocket was determined to be located ***5.19 inches*** from the base theoretically.

3. Moment of Inertia Calculation

The moment of inertia for the nose cone is found using the equation:

$$I = \frac{3}{10}mr^2 = \frac{3}{10}(.000514slug)(.967in)^2$$

$$= 3.60\text{x}10^{-5}\ \text{slug·in}^2 \quad (16)$$

a) The body (cylinder):

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(.000891slug)(.967in)^2$$

$$= 1.04\text{x}10^{-4}\ \text{slug·in}^2 \quad (17)$$

b) The engine (cylinder):

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(.000123slug)(.338in)^2$$

$$= 7.03\text{x}10^{-6}\ \text{slug·in}^2 \quad (18)$$

c) The fins (trapezoid):

$$I = \frac{m}{3}(bh)^2 = \frac{0.00069slug}{3}(0.736in)(1.125in)$$

$$= 0.000189\ \text{slug·in}^2 \quad (19)$$

The parallel axis theorem must be used:

$$I = I_{cm} + md^2 = 0.000563\ \text{slug·in}^2 \quad (20)$$

The summation of this data is represented by equation 4, which gives:

$$\boldsymbol{I = 8.99\text{x}10^{-4}\ \text{slug·in}^2} \quad (21)$$

4. Payload Calculation

A diagram showing the mission profile is provided below in order to obtain a better understanding of the calculations involved.

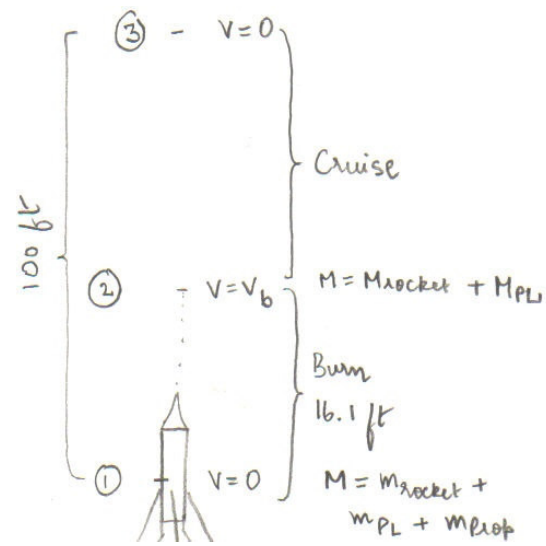

Figure 5 (mission profile)

We first analyze the motion of the rocket from position (1) to position (2) on the figure 5. We use the equations of motion between these two points knowing that the rocket engine burn time is 0.9 seconds from graph (1), in order to calculate the burn height s_b.

From graph 2 we obtain the average thrust generated by the engines to be 1.314 lbs and the corresponding average acceleration is 41.6 ft/s^2. Using the equations of motion, we see that the burn height s_b is given by,

$$s_b = \tfrac{1}{2}(41.6)(.9)^2 = 16.1 \text{ ft} \qquad (22)$$

Thus, the distance the rocket has to cruise unassisted by power is 83.9 ft. By the law of conservation of energy, the kinetic energy at burnout must be approximately equal to the potential energy gained until maximum height is reached. We obtain the following relationship,

$$\tfrac{1}{2} V_b^2 = (32.2 \text{ ft/s}^2)(83.9 \text{ ft}) \qquad (23)$$

From this we obtain the burnout velocity to be 73.5 ft/s.

We then plug the burnout velocity value into equation 12 and obtain the following relationship where m_{PL} is the payload mass,

$$v_b = (32.2\, ft/s)(73.71\, lb \cdot s / lb_m)\ln\left(\frac{63.3 + mpl}{57.06 + mpl}\right) \qquad (24)$$

On simplification of the above equation, the payload mass works out to be **141.24 g.**

VI) RESULTS

This section is divided into two parts. We will first summarize the various theoretical results obtained from the calculations.

Table 2 (Theoretical Results)

No.	Component	Value
1	Moment of Inertia	.000189 slugs.in^2
2	Center of gravity	5.19 inches
3	Payload	100 grams

It is important to note that the payload of 100g that was used in the experiment differed from the theoretically calculated value of 141.24g. This is because of 2 main reasons. Firstly, the mass of glue used was not taken into account but was later measured to be about 10 grams. Also the drag and wind factors were not taken into account. Reductions were made as necessary.

The experimental results obtained were as follows,

The actual center of gravity for the rocket was found by determining at what distance from the base one needs to suspend the rocket so that it balances, after it had been assembled. The distance was found to be 5.1 in. from the base.

While the rocket was launched, 2 stations located 100ft from the launch site measured the angle the rocket made with the horizontal at maximum height. This angle was found to be an average of 49º. The vertical distance was then found by taking advantage of the trigonometry, using the law of sines.

$$y = \frac{100\, ft}{\sin(90° - 49°)} \sin 49° = 115 \text{ ft} \qquad (25)$$

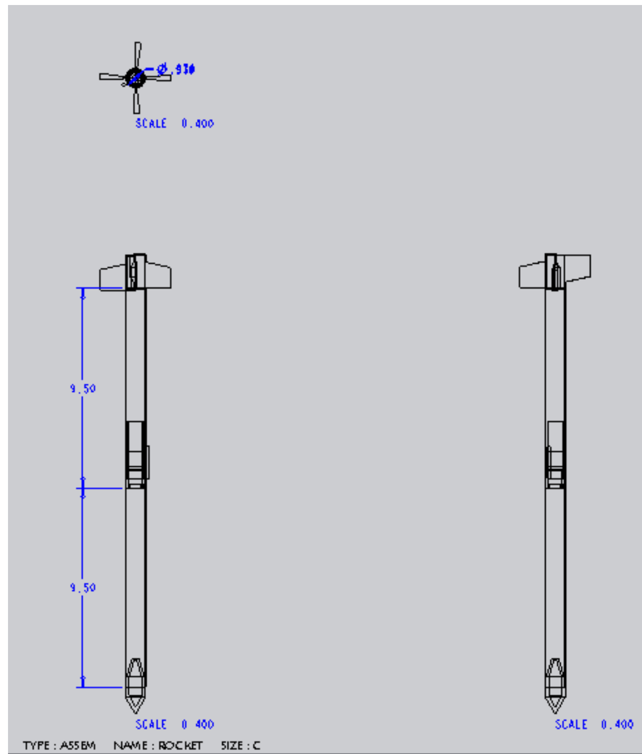

Figure 6 (Rocket 3-view drawing)

VII) DISCUSSION OF RESULTS

The rocket attained a height of 115 ft. as measured using the protractor apparatus and relevant trigonometry. This exceeded the required height to be attained by 15ft. or a factor of 15 %. It is important to note that this lies well within the error bounds for the measurement of the protractor angle.

The experimentally determined center of gravity was found to be at a distance of 5.1 inches from the base, which differs from the theoretical value of 5.19 inches, by 1.7 %.

Overall, these results obtained were satisfactory and the experiment was successful. This verifies the equations used for payload calculation. It is also important to note here that the actual payload inserted into the rocket was of mass 100g, which differed from the theoretically calculated value of 140.23grams. The reason this reduction was made was to account for neglected factors such as drag contribution, mass of glue and wind effects.

VIII) SOURCES OF ERROR

1. A primary source of error for this experiment was the fact that the effects of drag were virtually neglected and were only taken into account as a scaled factor. Had actual drag and wing effects been taken into consideration, the rocket might have attained the exact required height.
2. The mass of the payload used was in the form of penny coins and were weighed using a simple balance. Inaccuracies could exist in this measurement.
3. The measuring method for the rocket height involved the sighting of the point of maximum height and manually pointing and obtaining and angle, which inherently has errors mostly due to human factors.
4. The existence of wind effects altered the rocket from a perfectly horizontal motion to a curved path.
5. The theoretical values were determined assuming the engine used performed according to its specifications and similar to the test data engine. This may not necessarily be the case.

IX) CONCLUSIONS

This project was successful in helping the students better understand the effects of drag on a body in flight. The error found between the desired height reached by the rocket (100ft) and the actual height (115ft) was only 15%. This proves that the equations used for the data reduction are valid in this experiment. Additionally, the center of gravity that was experimentally determined conformed well with the theoretical calculation. Any error sources are listed in the relevant section. A parasitic drag coefficient for the rocket was also determined to study the effects of drag on the rocket motion. Drag was neglected in the calculations but was taken into account in a percent reduction of the actual payload used when compared to the theoretically determined payload. The most beneficial part of this experiment was the preparation taken: the calculations, and building the rocket. One change that the students wish to see is a better use of the helium balloon that was suppose to reach 100ft that would provide a better benchmark against which we can measure the height of the rocket.

X) REFERENCES

a) Shevell, Richard S. Fundamentals of flight 1989

b) Anderson, John D. Introduction to flight 2005

c) ESTES® Model Rockets

http://www.estesrocket.com

d) American Institute of Aeronautics and Astronautics (AIAA) http://aiaa.com

e) ProEngineer Wildfire® (Please see attached drawings)

A) ProEngineer Wildfire ® scaled drawing of the rocket

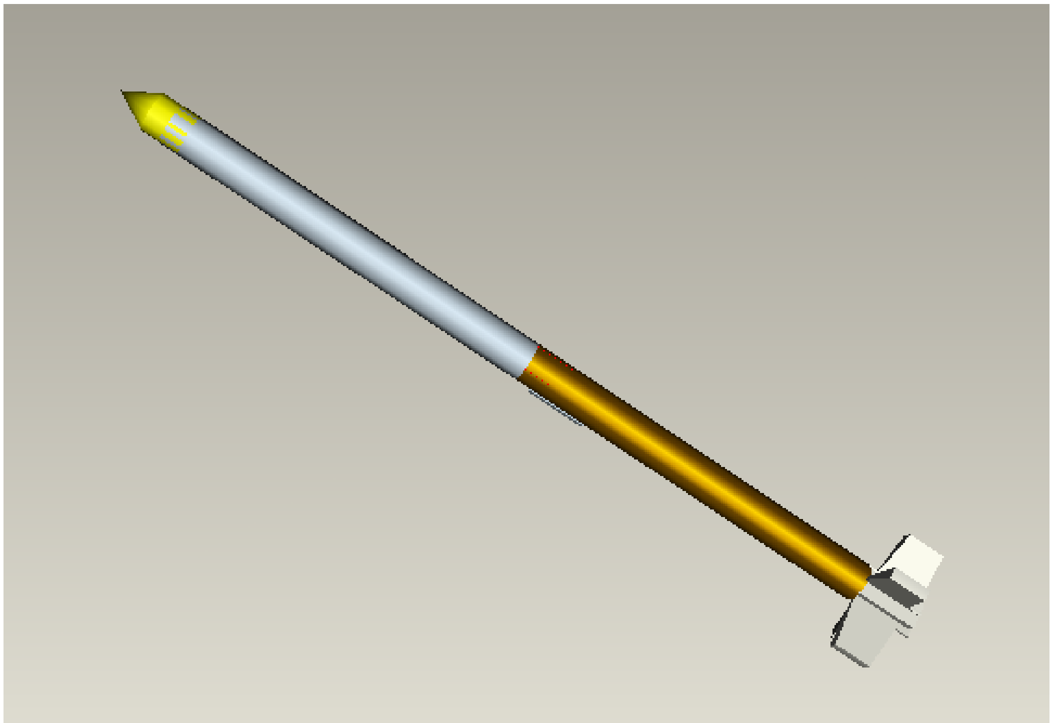

Figure 7 (Rocket scaled drawing)

B) ProEngineer Wildfire ® exploded view of the rocket showing parts

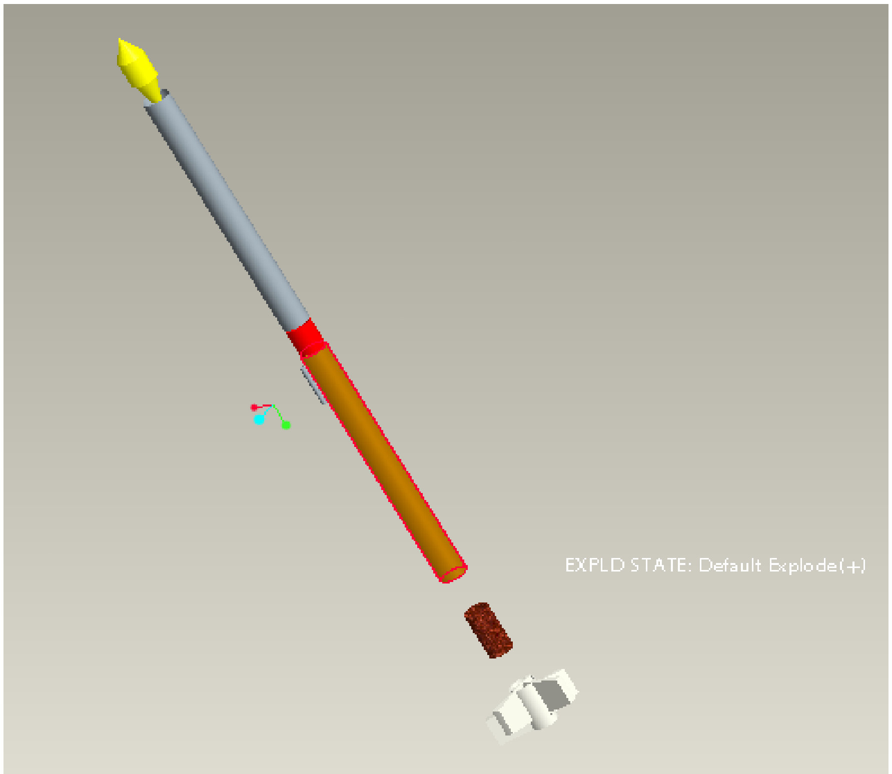

Figure 8 (Exploded View)

C) Three view drawing of the rocket

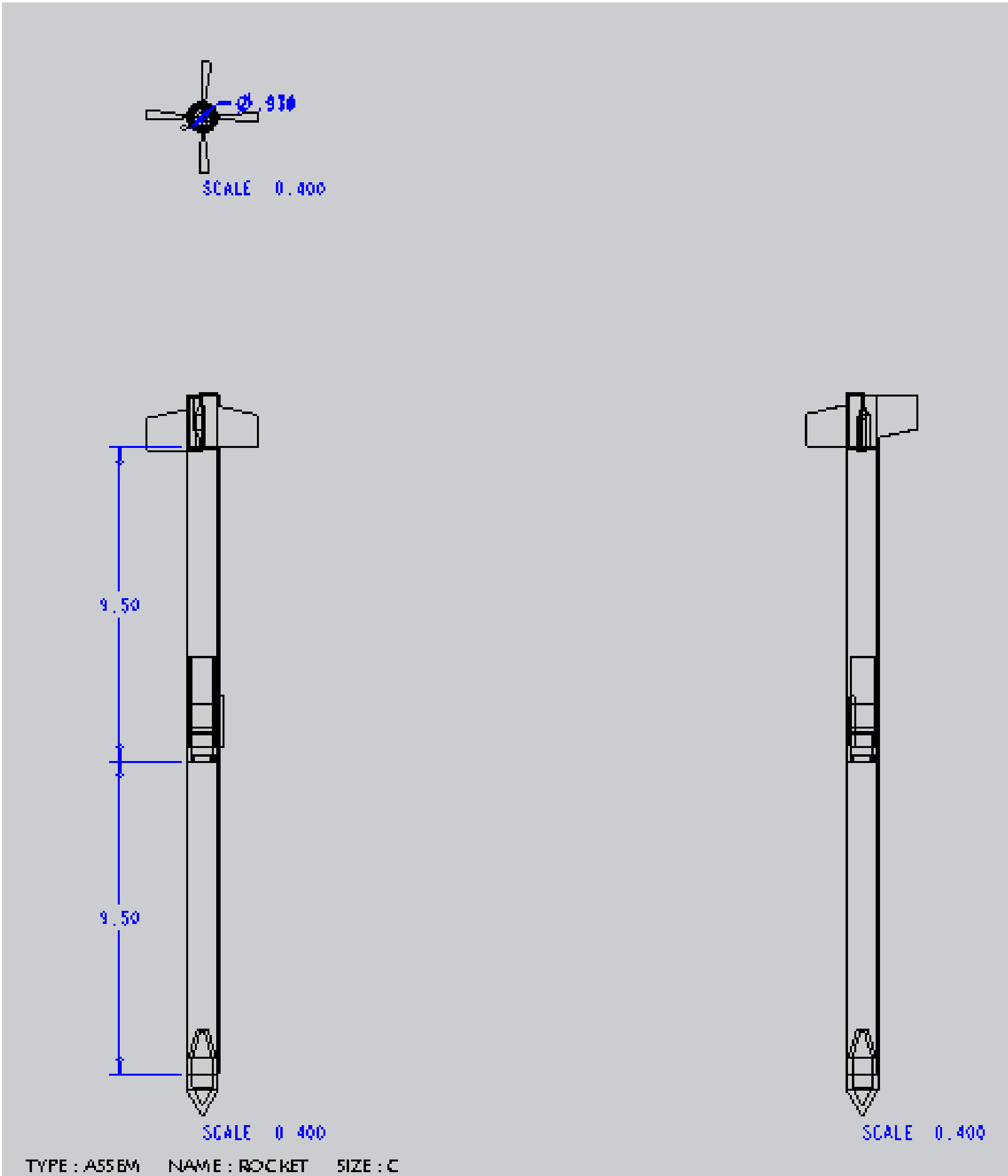

Figure 9 (Rocket 3 view)